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Improving the performance of existing missile autopilot using simple adaptive control

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SUMMARY

A simple add-on adaptive control algorithm is presented. The paper demonstrates via example that the performance of existing missile autopilot can be improved. The algorithm involves the synthesis of parallel feedforward, which guarantees that the controlled plant is almost strictly positive real. It is proved in the paper that such a parallel feedforward always exists. The proof is based on the parameterization of a set of stabilizing controllers. This parameterization enables straightforward design and implementation of the add-on simple adaptive control algorithm. Copyright © 2014 John Wiley & Sons, Ltd.

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KEY WORDS: simple adaptive control; missile autopilot; ASPR system synthesis

1. INTRODUCTION

Very often, the designer of an existing autopilot system is required to improve the performance under changing and uncertain environments. The improved performance may include larger performance envelope, faster response, improved robustness, and so on.

There are several approaches to confront this challenge. Among them are the following: (i) redesign the entire autopilot; (ii) replace or add sensors; (iii) adopt new control technology; and (iv) apply new algorithms with the existing hardware. In this paper, the proposed approach is to use an add-on adaptive control algorithm. The main advantage of the proposed approach is that the autopilot is not completely redesigned, rather the original system is preserved, and a simple addendum improves the performance and robustness.

The novelty of this paper is twofold: (1) Existence of an algorithm that not only improves the performance and robustness of a given autopilot but also adapts the control system to eventual parameter changes; and (2) synthesis of the algorithm for proper and stable plants and also for any strictly proper plant. The synthesis deals with SISO systems and is based on the parameterization of a set of stabilizing controllers. This parameterization enables straightforward design and implementation of the add-on simple adaptive control (SAC) algorithm [1–3]. Gain conditions and convergence of SAC are discussed in [4], where it is shown that the SAC adaptive feedforward gains perform a steepest decent minimization of the error without using any specific knowledge on plant parameters. An updated reference for the theory of passivity and adaptive control in nonlinear systems is in [5].

Promising results of add-on SAC in missile application appear in [6]. As an example, the add-on adaptive control algorithm is applied to improve the performance and robustness of the 'three-loop autopilot' [7–9]. Similar approach has been used in [10–13].

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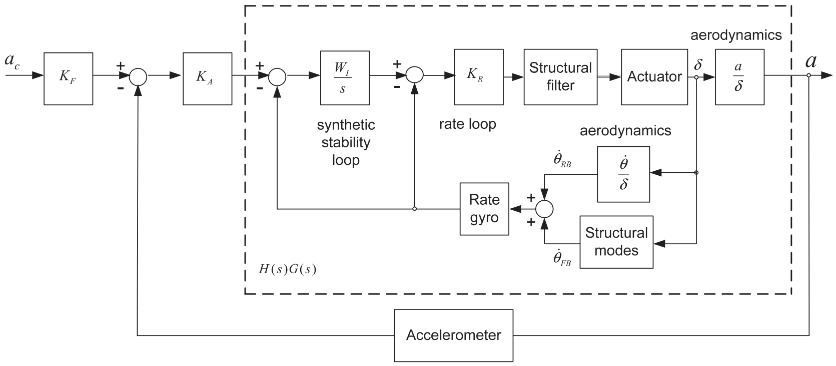

Figure 1. Three loop pitch/yaw autopilot.

The paper states the problem in Section 2, reviews the SAC algorithm in Section 3, and describes the add-on SAC algorithm in Section 4.The application of the add-on SAC algorithm to existing design of three loop autopilot is presented in Section 5. Conclusions are given in Section 6. The existence and synthesis of parallel feedforward that converts SISO system to be almost strictly positive real (ASPR) are discussed in the Appendix.

2. STATEMENT OF THE PROBLEM

Most air to air and surface to air missiles use a three-loop lateral autopilot [7], as shown in Figure 1. The three loops are the synthetic stability loop that includes the integrator, the accelerometer feedback loop, and the rate loop. The major components of the flight control system include the airframe, the aerodynamic control surfaces, the actuator, the rate gyro, and the accelerometer. The rate loop is the wideband loop, and structural vibrations are most evident in that loop.

The design scheme presented in Figure 1 is based on the following assumptions:

1. The Inertial Measuring Unit is located in the body center of gravity.
2. Each of the modal transfer functions is in parallel with the rigid body aerodynamic transfer function. The gyro senses the flexible body motions in addition to the rigid body motions.
3. The accelerometer also measures vibration of the body; however, this additional path has much less effect on system response and therefore, has been neglected.
4. The accelerometer transfer function is assumed to be 1, that is, ideal accelerometer.

The problem considered here is the design of an autopilot with improved performance and robustness based on existing flight control design. The proposed design uses add-on SAC.

3. REVIEW OF THE SAC ALGORITHM

The SAC is a special adaptive model following methodology that requires the plant, which could be of a very large dimension, to track a model, without requiring the model to be of the same order as the plant [1, 2]. The model could be a first-order low-pass filter incorporating only the desired plant time constant or a linear system just large enough to generate a general desired command. Even more important, basically, the resulting adaptive controller is of the same order as the reduced-order model. The SAC methodology assumes that some prior knowledge on the stability or the stabilizability properties of the plant to be controlled is available. This basic knowledge can be used to test the ASPR properties of the plant or to build the proper parallel feedforward that can render the plant ASPR and make the use of SAC safe and robust (see [1] and [14]). The SAC can be used in an attempt to further improve performance and to maintain the desired performance over the entire range of uncertainty. The SAC scheme is described in Figure 2.

The scheme contains the reference model to be followed, the adaptive gains, and the parallel feedforward $D(s)$, which ensures that the augmented plant is ASPR.

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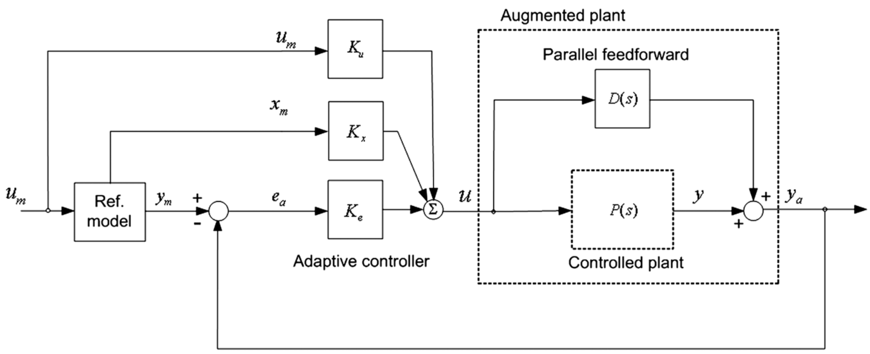

Figure 2. Simple adaptive control (SAC) scheme.

The reference model is represented by

$$\begin{aligned} \dot{x}_m(t) &= A_m x_m(t) + B_m u_m(t) \\ y_m(t) &= C_m x_m(t) \end{aligned} \tag{1}$$

The adaptive gains are obtained as a combination of 'proportional' and 'integral' gains and satisfy the following equations:

$$\begin{aligned} K_e(t) &= K_{Pe}(t) + K_{Ie}(t) \\ K_x(t) &= K_{Px}(t) + K_{Ix}(t) \\ K_u(t) &= K_{Pu}(t) + K_{Iu}(t) \end{aligned} \tag{2}$$

$$\begin{aligned} K_{Pe}(t) &= e_a e_a^T \Gamma_{Pe} \\ K_{Px}(t) &= e_a x_m^T \Gamma_{Px} \\ K_{Pu}(t) &= e_a u_m^T \Gamma_{Pu} \end{aligned} \tag{3a}$$

$$\begin{aligned} \dot{K}_{Ie}(t) &= e_a e_a^T \Gamma_{Ie} - \sigma K_{Ie}(t), \quad K_{Ie}(0) = 0, \\ \dot{K}_{Ix}(t) &= e_a x_m^T \Gamma_{Px} \\ \dot{K}_{Iu}(t) &= e_a u_m^T \Gamma_{Iu} \end{aligned} \tag{3b}$$

where all the Γ's are positive definite matrices of proper dimension, and σ is a positive scalar.

The control u is given by

$$u = K_u u_m + K_x x_m + K_e e_a \tag{4}$$

The basic parallel feedforward idea is quite simple [1]: if a system $G(s)$ is known to be stabilizable thorough a controller $H(s)$, then the inverse $H^{-1}(s)$ used in parallel with the original plant makes the augmented system $G_a(s) = G(s) + H^{-1}(s)$ minimum phase. The original $G(s)$ could be both unstable and non-minimum phase. When the final relative degree is 1 (in SISO systems as well as in $m \times m$ multivariable systems), the resulting augmented system is ASPR. (Note: Relative degree has customarily been defined as the number of differentiations, k, that the output equations $y = Cx$ must pass before one obtains a relation of the form $y(k) = Mx + Nu$ where u explicitly shows. Therefore, although in SISO systems relative degree, 1 implies n poles and $n - 1$ zeros; in m×m multivariable systems, it implies n poles and $n - m$ zeros). The poles of the augmented system $G_a(s)$ are not affected by the use of parallel feedforward and could be unstable, but $G_a(s)$ can now be safely used with SAC, and the stability of the system and the asymptotic tracking of the resulting adaptive control system are guaranteed.

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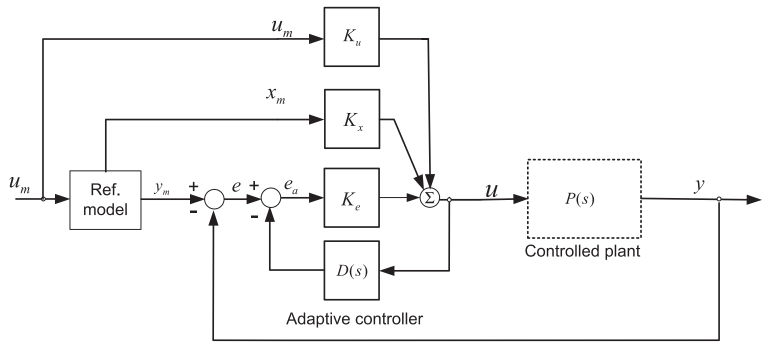

Figure 3. Equivalent simple adaptive control scheme.

The SAC scheme can be reconfigured by using the following relations:

$$\begin{aligned} e &= y_m - y \\ e_a &= y_m - y_a = y_m - (y + Du) = (y_m - y) - Du \\ e_a &= e - Du \end{aligned} \tag{5}$$

The equivalent scheme is depicted in Figure 3.

4. THE PROPOSED SOLUTION OF THE PROBLEM – AN ADD-ON ADAPTIVE CONTROLLER

The following reasons justify the integration of the adaptive controller in the existing closed-loop design. (a) The original system is not changed (this is good for maintenance; the existing personnel is familiar with the equipment). (b) The uncertainty of the closed loop is reduced by the existing controller, therefore the added controller have to deal with less uncertainty. (c) The added controller can always be disconnected, and the original performance is restored.

To efficiently integrate the SAC algorithm in the existing design of the autopilot, a different configuration of Figure 1 is needed. The respective architecture is presented in Figure 4. See [15] for the nomenclature.

The closed-loop transfer function is

$$P_0 = \frac{K_A HG}{1 + K_A HG} \tag{6}$$

In order to proceed and present the proposed add-on adaptive controller, the two blocks controller configuration of Figure 4 is modified. The equivalent block diagram is presented in Figure 5.

Following Figure 5, the new closed-loop transfer function is

$$\tilde{P} = \frac{K_1 K_2 HG}{1 + (1 + K_1) K_2 HG} \tag{7}$$

The configurations of Figures 4 and 5 are equivalent if $(1 + K_1)K_2 = K_A$ and $K_F P_0 = \tilde{K}_F \tilde{P}$.

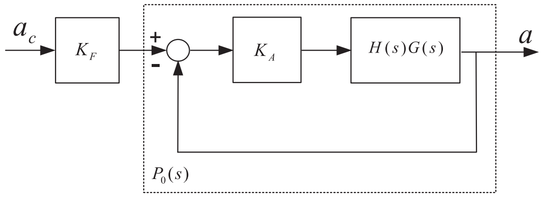

Figure 4. Two blocks controller architecture derived from Figure 1.

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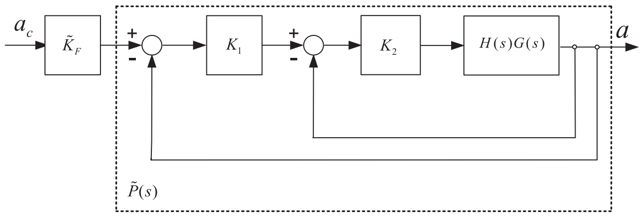

Figure 5. Equivalent three blocks controller configuration.

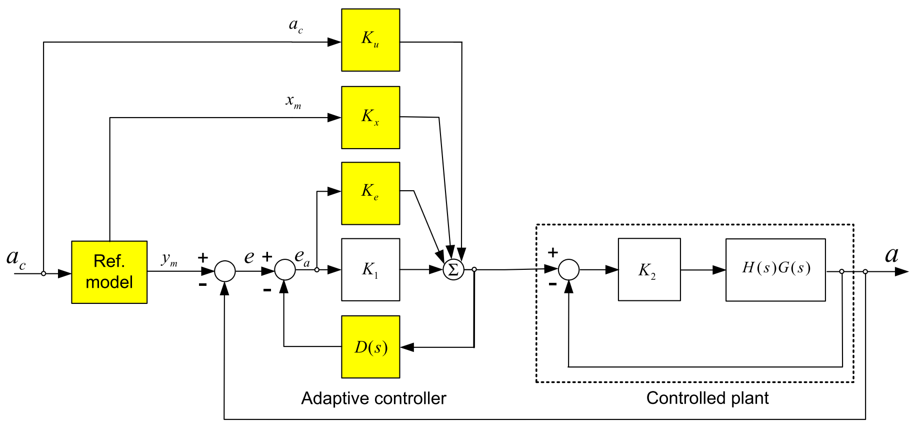

Figure 6. Three loop autopilot with add-on adaptive controller.

If $(1 + K_1)K_2 = K_A$, then $\tilde{P}$ is a stable transfer function with the same closed-loop characteristics as the original transfer function P_0. In this case, the only difference between the systems $K_F P_0$ and $\tilde{K}_F \tilde{P}$ is the low frequency gain. The requirement $K_F P_0 = \tilde{K}_F \tilde{P}$ leads to the relation $\tilde{K}_F = \frac{K_F K_A}{K_1 K_2}$.

Because $K_A/K_2 = 1 + K_1$ and $K_A > 0$, $K_1 > 0$, $K_2 > 0$, then $K_2 < K_A$. The reduced gain ensures quiet response without noise amplification when maneuvers are not needed. In case of maneuvers, the proposed add-on adaptive controller will increase the gain in the required time intervals, provided that stability is guaranteed via a matched parallel feedforward. The proposed add-on control architecture is based on SAC and presented in Figure 6.

The following points are important:

1. $(K_e)_{eq} = K_e + K_1$ is bounded from below by K_1 where $K_{Ie}(0) = 0$.
2. The gain $\tilde{K}_F$ is included in the adaptive gains used to follow the reference model.
3. If the reference model is a first-order system of the form

$$\frac{y_m}{u_m} = \frac{1}{1 + s\tau}$$

that is,

$$\dot{x}_m = -\frac{1}{\tau} x_m + \frac{1}{\tau} u_m$$

then

$$x_m \equiv y_m$$

In this case, the time constant τ is included in the tunable gains Γ_{Pu}, Γ_{Iu}.

The design of the SAC controller involves the selection of K_1, K_2 and the synthesis of $D(s)$. Some synthesis procedures of $D(s)$ are discussed in Appendix. The mainly used procedure is cited

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below and is valid for a proper and stable plant. This is our case because the controlled plant $K_2HG/(1+K_2HG)$ is stable.

The selection process is detailed in the following text.

1. Choose $K_1 >> 1$ and K_2 which satisfy $(1+K_1)K_2 = K_A$.
2. Replace K_1 in Figure 5 by a stabilizing controller $C(s)$ such that:
   (a) $C(s)$ is stable.
   (b) $C(s)$ is a minimum phase (all the zeros are in the left-hand plane (LHP)).
   (c) $C(s)$ has high gain.
   (d) $C(s)$ has relative degree 0 or -1 (zero excess of 0 or 1).
3. Select $D(s) = C^{-1}(s)$

Remarks

1. If $K_1 >> 1$ then $\tilde{P} \approx P_0$.
2. The requirement of high gain for $C(s)$ comes from the relation $D(s) = C^{-1}(s)$. Because the SAC algorithm controls the augmented error $e_a(t)$ and the true tracking error is $e(t)$, the gain of $D(s)$ is required to be as small as possible.
   Because $K_1 >> 1$, the stabilizing controller $C(s)$, which replaces K_1 can be designed with high gain.
3. The controlled plant $K_2HG/(1+K_2HG)$ is less sensitive to parameter variations. Therefore, the synthesized $D(s)$ will stand parameter variations and will guarantee that the controlled plant with the parallel feedforward is ASPR.
4. The preceding procedure provides sufficient conditions for the augmented plant to be ASPR.

5. EXAMPLE

In this example, the add-on SAC based control algorithm is applied to the three-loop missile autopilot [7–9].

It is demonstrated that the application of the add-on SAC based controller at single operating point in the flight envelope significantly improves the performance of the autopilot. Observe that the use of the proposed add-on SAC controller does not eliminate the use of gain scheduling within the whole flight envelope. The example considers two cases. Both cases use the same autopilot nominal design but with different changes in the nominal aerodynamic transfer functions:

(i) Case 1 – the nominal gain of the aerodynamic transfer functions a/δ is replaced by constant gains: nominal $+6$ dB and nominal -6 dB.
(ii) Case 2 – the nominal gain of the aerodynamic transfer functions a/δ and $\dot{q}/\delta$ is changed abruptly after 2 s by -6 dB.

Remarks
Changes of ± 6 dB in the gains are large changes that challenge the gain margin of the nominal design.

5.1. Existing three-loop autopilot data and performance

The autopilot parameters follow Nesline and Nesline [7]. The relevant gains and transfer functions follow the notation of Figure 1.

Controller gains:

$$K_F = 1/0.7583$$

$$K_A = 0.0018208$$

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$$W_I = 15.82481$$

$$K_R = 0.33835$$

Aerodynamic transfer functions:

$$\frac{\dot{\theta}}{\delta} = \frac{0.6477(0.676s + 1)}{(s/22.4)^2 + 2 \times (0.052/22.4)s + 1}$$

$$\frac{a}{\delta} = \frac{-1116.5(-0.00081s^2 + 0.0010546s + 1)}{(s/22.4)^2 + 2 \times (0.052/22.4)s + 1}$$

Rate gyro transfer function:

$$\frac{\dot{\theta}_G}{\dot{\theta}} = \frac{1}{(s/500)^2 + 2 \times (0.65/500)s + 1}$$

Actuator transfer function:

$$\frac{\delta}{\delta_c} = \frac{1}{(s/250)^2 + 2 \times (0.7/250)s + 1}$$

The transfer functions from control surface angle to body rate for each of the flexible modes are the following:

First structural mode:

$$\frac{\dot{\theta}_{FB_1}}{\delta} = \frac{0.00134[(s/345)^2 + 1]s}{(s/259)^2 + 2 \times (0.015/259)s + 1}$$

Second structural mode:

$$\frac{\dot{\theta}_{FB_2}}{\delta} = \frac{0.000664[(s/255)^2 + 1]s}{(s/6999)^2 + 2 \times (0.022/699)s + 1}$$

Structural filters transfer functions:

$$F_1(s) = \frac{(s/259)^2 + 1}{(s/259)^2 + 2 \times (0.7/259)s + 1}$$

$$F_2(s) = \frac{(s/699)^2 + 1}{(s/699)^2 + 2 \times (0.7/699)s + 1}$$

5.1.1. Case 1 – Different constant gains. The first case (Case 1) considers three different gains of the aerodynamic transfer function a/δ: nominal, +6 dB and −6 dB. Typical step responses of the design described in [7] are depicted in Figure 7. The step responses include the nominal case and cases where the gain of the transfer function a/δ is different from the nominal by ±6 dB. The response demonstrates the performance and robustness of the design as presented in [7]. The response is clearly non-minimum phase with nominal time constant of about 250 ms and with small amplitude oscillations. Significant changes of the response are observed.

5.1.2. Case 2 – Abrupt gain change. In this case, the nominal gains of the two aerodynamic transfer functions a/δ and $\dot{q}/\delta$ are changed abruptly after 2 s by −6 dB. The step responses for the nominal gain and for the abruptly changing gain are depicted in Figure 8. Large oscillations and slower time response are clearly observed after the gain abrupt change.

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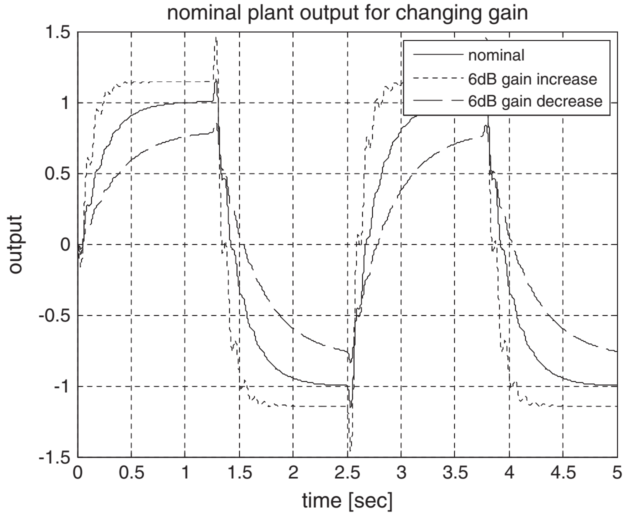

Figure 7. Step response of the existing three-loop autopilot design in the nominal case and where a/δ gain is different from the nominal by ± 6 dB.

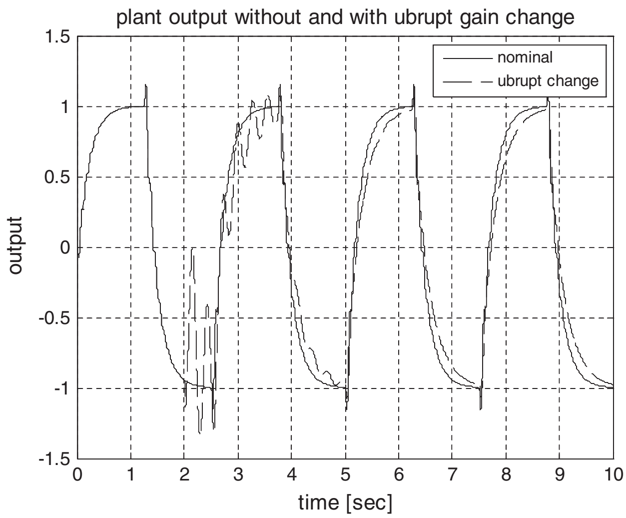

Figure 8. Step response of the existing three-loop autopilot design in the nominal case where a/δ and $\dot{q}/\delta$ gains are changed abruptly after 2 s by -6 dB.

5.2. Add-on SAC based adaptive controller data and performance

The equivalent three-loop autopilot gains, as defined in Figure 5, are

$$K_1 = 2.6414$$
$$K_2 = 5 \times 10^{-4}$$

The synthesized parallel feedforward transfer function is

$$D(s) = C^{-1}(s) = \frac{1}{35} \frac{s/2.5 + 1}{s/10 + 1} \tag{8}$$

No special care has been taken in the design of the stabilizing controller in this example. The design of a stabilizing controller with relative degree 0 or -1, which meets additional requirements, is beyond scope of this paper. Robustness maximization with respect to changes in the airframe parameters is an example of an additional requirement.

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The selected model transfer function reflects the requirement to decrease the time constant of the autopilot response to 150 ms.

The implemented reference model transfer function is

$$M(s) = \frac{1}{0.15s + 1} \tag{9}$$

The designed adaptive controller gains are

$$\begin{aligned} &\Gamma_{Pe} = 10^3 \quad \Gamma_{Ie} = 10^4\,,\ \sigma = 10^{-3} \\ &\Gamma_{Px} = 10^1 \quad \Gamma_{Ix} = 10^2 \\ &\Gamma_{Pu} = 10^1 \quad \Gamma_{Iu} = 10^2 \end{aligned}$$

5.2.1. Nominal gain performance with the add-on adaptive controller. Figure 9 depicts the nominal response of the existing three-loop autopilot design versus the response of the proposed add-on adaptive controller. As it can be seen, the add-on SAC response reaches the 95% of the steady state value faster than the original autopilot response, although the time constant of both is the same, about 250 ms. This demonstrates that the add-on adaptive controller is an effective tool in improving the performance of an existing autopilot.

5.2.2. Non-nominal Gain performance with the add-on adaptive controller. Figure 10 shows the output of the reference model y_m and the output of the plant y with the proposed add-on adaptive controller. The response is given for the nominal a/δ gain, and where, the gain is different from the nominal by ± 6 dB. It is clearly shown that the performance has been preserved in spite of the large change ± 6 dB.

The variation of the time constant and the DC gain can be observed by comparing Figure 10 (the existing autopilot with the add-on adaptive controller) to Figure 7 (the existing autopilot alone). With the add-on adaptive controller the variations are much less than those observed in the nominal autopilot.

By using the add-on SAC scheme, the controlled output is y_a and not y. Hence, there is a small penalty in the tracking error. The resulting tracking errors can be observed in Figure 11. One can see that a small additional error due to the parallel feedforward is a small price to pay for guarantees of robustness of the adaptive controller and thus, for the substantial performance improvement over the linear controller.

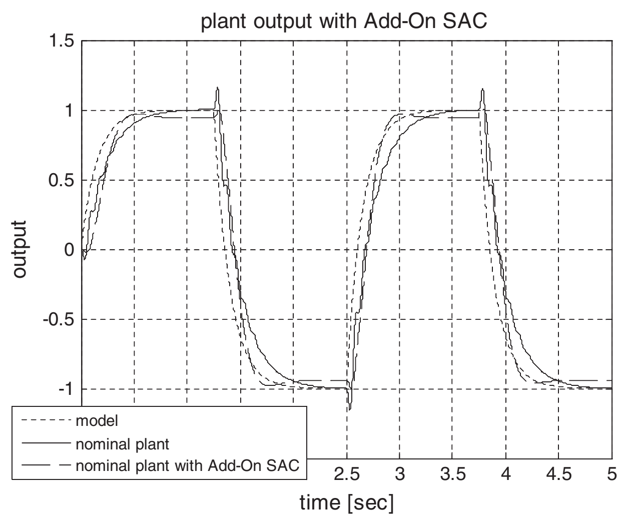

Figure 9. Nominal step response of the existing three-loop autopilot design versus the nominal step response of the proposed add-on adaptive controller.

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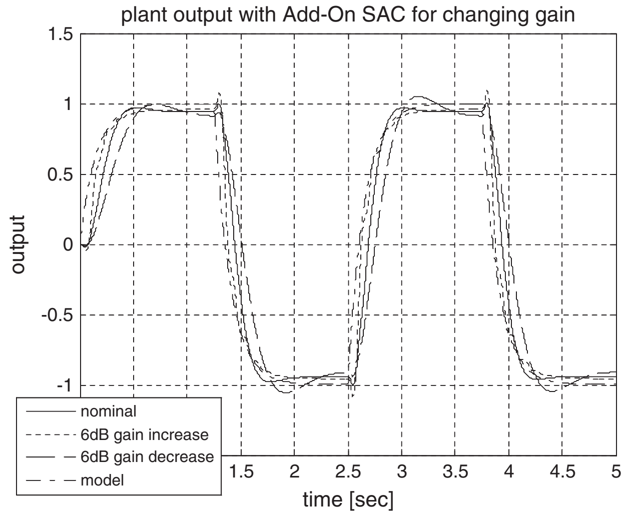

Figure 10. The model output y_m and the plant output y in the nominal case, and where, a/δ gain is different from the nominal by ± 6 dB. The reference model is $M(s) = 1/(0.15s + 1)$.

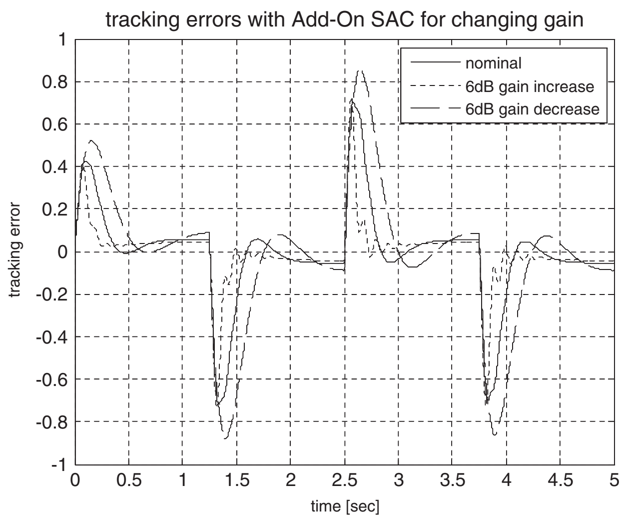

Figure 11. The tracking error with add-on SAC in the nominal case, and where, a/δ gain is different from the nominal by ± 6dB.

Figure 12 shows the add-on adaptive controller gains. It also must be noted, however, that the adaptive gain $K_{Ie}(t)$ in (3b) would increase whenever the tracking error is not zero. Although it is proved [1, 2] that all adaptive gains converge to constant finite values under ideal conditions, the gain $K_{Ie}(t)$ without the sigma term in (3b) would continually increase in the presence of any noise, even at those noise levels that are negligible for any other practical purposes. In spite of the fact that an ASPR system remains stable with arbitrarily high gains, these gains may be too high for any practical (and numerical) purpose and may even diverge in time. This is the reason for the adoption of Ioannou's simple idea [16, 17] and addition of a σ-modification term (or forgetting factor) in (3b). With the σ-modification term, the error gain (i.e., the main adaptive loop gain) increases whenever the error tends to increase and decreases when large gains are not needed any more. The coefficient σ can be very small, because its aim is only to prevent the gain from increasing without bound. This effect is not felt by the control gains $K_{Ix}(t)$ and $K_{Iu}(t)$. The variation in the adaptive gains due to ± 6 dB change in the a/δ gain is not more than 1.5 times the nominal value.

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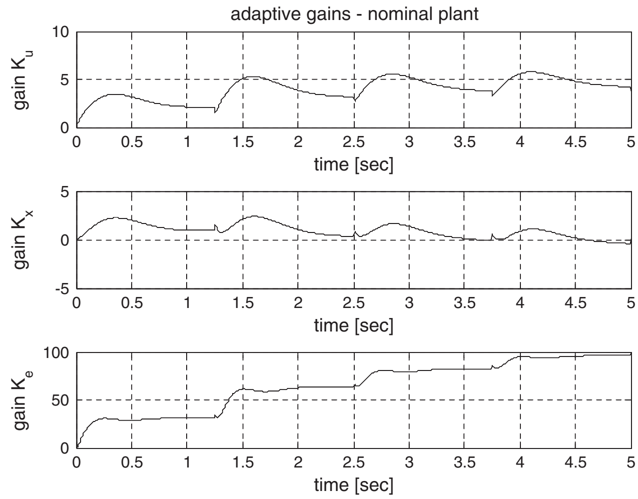

Figure 12. The gains of the add-on simple adaptive control algorithm.

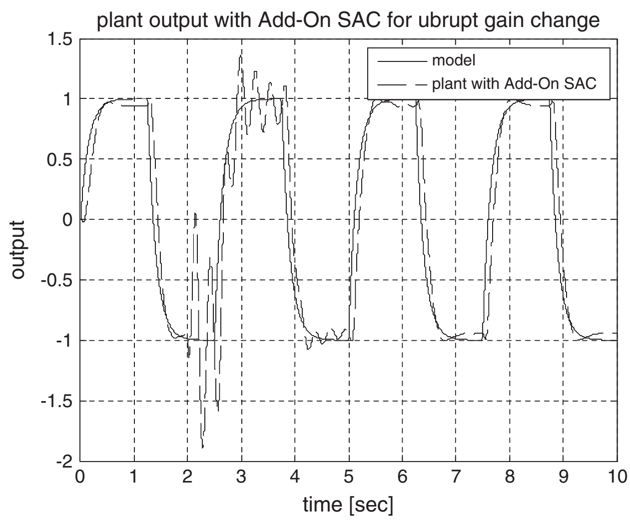

Figure 13. The model output y_m and the plant output y for the abruptly changing gain with add-on SAC.

5.2.3. Abrupt gain change performance with the add-on adaptive controller. The abrupt gain change performance is depicted in Figures 13–15. Figure 13 describes the model response and the response obtained with the add-on SAC adaptive controller. Figure 14 presents the associated tracking error. The gains of the SAC adaptive algorithm are presented in Figure 15.

Remarks

1. Note that the paper compares a classical design, where the input command is the only signal that is supplied to the plant, with the add-on SAC, where the command is first passed through the model and then the output of the model is supplied to the plant as the new desired command along with the feedforward signals. One is therefore tempted to use the same reference model with the classical design. However, the first effect of using the model would be slowing the plant response even more. In order to improve the time response, one could then think of using fixed feedforward gains that, in principle, could result in prefect model tracking without

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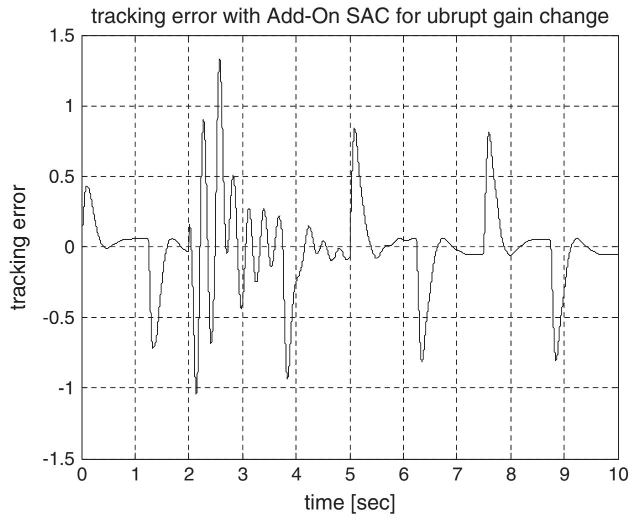

Figure 14. The tracking error for the nominal design and for the abruptly changing gain with add-on simple SAC.

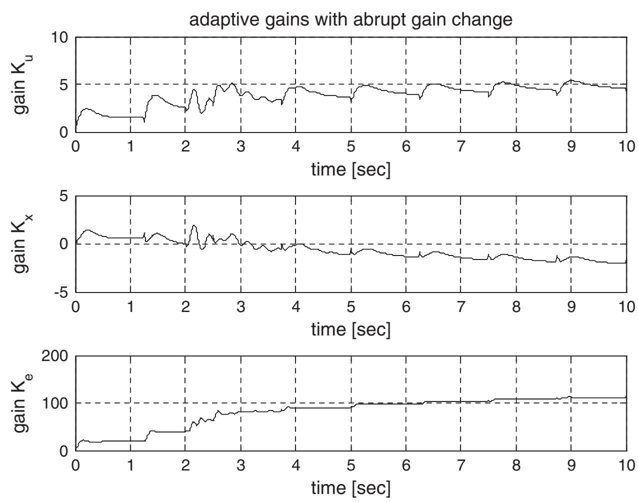

Figure 15. The gains of the add-on SAC algorithm for the case of abruptly changing gain.

requiring any adaptive controller. However, the reason fixed feedforward gains are not customarily used in missile design is because their computation would require *perfect* knowledge of plant parameters. Unlike the feedback loop that can stand some deviation from the nominal gain, the feedforward addition is entirely open-loop and, therefore, any deviation from the nominal values is transmitted as 100% error to the output. Instead, the adaptive feedforward gains are generated using the tracking error, and the adaptations ends when the error reaches its minimum.

2. At first look, one may find it peculiar to expect from an adaptive controller, which first must pass an adaptation process, to improve the performance of a well-designed fixed gain controller. The answer to this remark is that, although the adaptive feedforward algorithm was first of all developed from reasons of stability, it actually is the optimal control solution based on the steepest descent minimization [4]. In other words, from the very first moment and independently of any other factors, the adaptive feedforward gains perform a steepest descent minimization of the (square) tracking error.

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3. The next question is why would the internal adaptive gain be needed, when the LTI design has already reached the 'best' value? The answer is that whenever, because some sudden maneuver, the tracking error $e_y(t)$ attempts to increase, the internal adaptive gains also starts increasing at the rate of $e_y^2(t)$, and thus, because stability is guaranteed, forces the error to decrease. Furthermore, it only increases as long as it is needed and, due to the σ-term in (3b), it comes back down as soon as the need has passed.
4. Last but not the least, we assumed that a gain drop of 6 dB from the nominal value still remains within the stability domain. However, this is also only a design assumption and could also be affected by uncertainty. In any case, a gain drop that in the LTI design could lead to total destruction, may only tend to lead to some error increase with the add-on SAC, because it would result in sudden gain increase that would bring the plant back into the stability domain and, as test showed, would be hardly felt in the plant performance.

6. CONCLUSIONS

The add-on adaptive control algorithm that can always improve the performance of *any stable linear system* has been presented. It is shown and proved that such a controller always exists. The proofs are constructive, thus presenting a simple alternative where it is required to improve the system performance and robustness.

APPENDIX: EXISTENCE AND SYNTHESIS OF PARALLEL FEEDFORWARD THAT RENDERS SISO SYSTEM ASPR

Many readers and control practitioners express skepticism when they relate to the general idea of finding the appropriate parallel feedforward controller. Although after some experience, in most cases, finding an appropriate PFC is pretty straightforward; it is worth to give the necessary theory behind it in order to *prove* that a $D(s)$ *exists* and can be synthesized in the more general case.

Sufficient stability conditions for the SAC algorithm of Section 3 are discussed in [1, 2]. These conditions are briefly stated here: The add-on SAC algorithm is stable and the tracking error $e_a(t)$ asymptotically converges to zero if the parallel feedforward $D(s)$ is such that the augmented plant

$$P_a(s) = P(s) + D(s) \tag{A.1}$$

is ASPR. Positive real systems are discussed in [18]. Necessary and sufficient conditions for a system to be ASPR are stated below.

Theorem 1 ([1, 14]*)*
An LTI system is ASPR iff it is minimum phase and its relative degree is 0 or 1.

As will be shown later, it implies that if $D^{-1}(s)$ stabilizes $P(s)$, then $P_a(s)$ is a minimum phase. If in addition the relative degree of $P_a(s)$ is 1 or 0, the required ASPR conditions are satisfied.

The main result of this Appendix is that any minimal strictly proper system can be augmented in such a way that it is made ASPR. The first part of the Appendix deals with stable and proper systems. The second part extends the results to any minimal strictly proper system.

Proper and stable systems

Theorem 2 ([19]*, Section 5.1)*
Let S be the set of all stable, proper, and real-rational transfer functions. Assume that $P \in S$. Then the set of all controllers C, for which the feedback system is internally stable, equals

$$\{Q/(1 - QP) \quad ; Q \in S\} \tag{A.2}$$

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Proof
The full proof is presented in [19]. Here, only a sketch of the proof is given. If both Q and P are stable, then

$$C = Q/(1 - QP) \tag{A.3}$$

and the closed loop is

$$\frac{CP}{1+CP} = \frac{Q/(1-QP)P}{1+Q/(1-QP)} = \frac{QP}{1-QP+QP} = QP \in S \tag{A.4}$$

□

Lemma 1 ([20]*)*
Assume that $P \in S$, $Q \in S$. Let D be defined by $D^{-1} = C = Q/1 - QP$. Then $P + D$ is minimum phase.

Proof

$$P + D = P + (1 - QP)/Q = (QP + 1 - QP)/Q = 1/Q, \tag{A.5}$$

Because $Q \in S$ is stable then Q^{-1} is minimum phase. □

Lemma 2 ([20]*)*
Assume that $P \in S$, Q is stable and minimum phase, and $D^{-1} = Q/(1 - QP)$. Then, $P + D$ is minimum phase and stable.

Proof

$$P + D = Q^{-1}, \tag{A.6}$$

Because Q is stable and minimum phase, then Q^{-1} is minimum phase and stable. □

Theorem 3 ([20]*)*
Assume that $P \in S$, $Q \in S$ with relative degree {0,-1}, and $D^{-1} = Q/(1 - QP)$. Then $P + D$ is minimum phase with relative degree {0,1} and thus, ASPR.

Proof
The proof follows the lines of Lemmas 1 and 2. □

Theorem 4
Assume that $P \in S$, $Q \in S$ are minimum phase with relative degree {0,-1}, and $D^{-1} = Q/(1 - QP)$. Then $C = D^{-1}$ is minimum phase with relative degree {0,-1}.

Proof
Equation (A.6) leads to

$$P + C^{-1} = Q^{-1} \tag{A.7}$$

Let P, C, Q be described by

$$P = N_P/D_P \quad , C = N_C/D_C \quad , Q = N_Q/D_Q \tag{A.8}$$

where P is of order n , Q is of order n_Q , and the relative degree of Q is r. Then,

$$N_P/D_P \quad + D_C/N_C \ = D_Q/N_Q \tag{A.9}$$

and thus,

$$N_C/D_C \ = D_P N_Q/(D_P D_Q - N_P N_Q) \tag{A.10}$$

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Because $P \in S$, $Q \in S$, then D_P and D_Q have all roots in the LHP. Because $N_C = D_P N_Q$, then C is minimum phase.

Let $\rho(\cdot)$ and $\rho_r(\cdot)$ denote the degree and the relative degree of $(\cdot)$, respectively. Then,

$$\begin{aligned}\rho_r(C) &= \rho(D_C) - \rho(N_C) = \rho(D_P D_Q - N_P N_Q) - \rho(D_P N_Q) \\ &= \max\{n + n_Q,\ n + n_Q - r\} - (n + n_Q - r) = r = \rho_r(Q)\end{aligned} \tag{A.11}$$

If $\rho_r(Q) \in \{-1,\ 0\}$, then $\rho_r(C) = \rho_r(Q) \in \{-1, 0\}$ □

Remarks

1. The use of $D = C^{-1}$ where C is minimum phase with relative degree {-1,0} guarantees that the augmented plant $P + D$ is ASPR.
2. A controller C which is stable, minimum phase and has relative degree {-1,0} also guarantees that the augmented plant $P + D$ is ASPR.

Strictly proper systems

Let the minimal strictly proper nth order SISO plant P be described as

$$P(s) = C(sI - A)^{-1}B \tag{A.12}$$

where A, B, C are the state space matrices. Then a coprime factorization of P is (see [19]):

$$P(s) = N/M \tag{A.13}$$

where

$$M(s) = F\,(sI - (A + BF))^{-1}\,B + 1 \tag{A.14a}$$

$$N(s) = C\,(sI - (A + BF))^{-1}\,B \tag{A.14b}$$

$$X(s) = F\,(sI - (A + HC))^{-1}\,H \tag{A.14c}$$

$$Y(s) = -F\,(sI - (A + HC))^{-1}\,B + 1 \tag{A.14d}$$

are stable transfer functions. The matrices $F \in R^{1xn}$, $H \in R^{nx1}$ are real matrices selected such that $A+BF$ and $A+HC$ are stable. The matrices M, N, X, and Y satisfy the constraint

$$N(s)X(s) + M(s)Y(s) = 1 \tag{A.15}$$

Theorem 5 ([19]*, Section 5.4)*
Let the system $P = N/M$ be minimal strictly proper (not necessarily stable or minimum phase) and N, M be a coprime factorization over S (i.e., N, M are stable). Let X, Y be two transfer functions in S satisfying $NX + MY = 1$.

Then the set of all controllers, C, for which the feedback system is internally stable equals

$$\{(X + MQ)/(Y - NQ) \quad ; Q \in S\} \tag{A.16}$$

Lemma 3
Assume that P is minimal strictly proper and $Q \in S$. Let D be defined by $D^{-1} = C = (X + MQ)/(Y - NQ)$. Then, $P + D$ is minimum phase.

Proof

$$P + D = \frac{N}{M} + \frac{Y - NQ}{X + MQ} = \frac{NX + MY + NMQ - NMQ}{M\,(X + MQ)} = \frac{1}{M\,(X + MQ)}, \tag{A.17}$$

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Because $Q, X, M \in S$, then $M(X + MQ)$ is stable (as S is closed under addition and multiplication). Thus, $1/M(X + MQ)$ is minimum phase. □

Theorem 6
Assume that P is minimal strictly proper, $Q \in S$ with relative degree {0,-1}, and $D^{-1} = (X + MQ)/(Y - NQ)$. Then, $P + D$ is minimum phase with relative degree {0,1} and thus, ASPR.

Proof
Following Lemma 3 $P + D$ is minimum phase. Therefore, it is only left to show that $\rho_r(P + D) \in \{0, 1\}$ where $\rho_r(P + D)$ denotes the relative degree of $(P + D)$.

Because

$$P + D = 1/M(X + MQ), \tag{A.18}$$

then an equivalent requirement is to show that $\rho_r[M(X + MQ)] \in \{-1, 0\}$.

Equation (A.14) indicates that M, N, X, Y are of order n where $\rho_r(M) = 0$, $\rho_r(N) = r_N$, $\rho_r(X) = r_X$. Assuming that Q is of order n_Q with $\rho_r(Q) = r$, then

$$\rho_r[M(X + MQ)] = \rho_r\left[\frac{(s^n + ...)}{(s^n + ...)}\left(\frac{(s^{n-r_X} + ...)}{(s^n + ...)} + \frac{(s^n + ...)}{(s^n + ...)}\frac{(s^{n_Q - r} + ...)}{(s^{n_Q} + ...)}\right)\right] \tag{A.19a}$$

Therefore,

$$\rho_r[M(X + MQ)] = n + n_Q - \max\left(n_Q + n - r_X, n + n_Q - r\right) \tag{A.19b}$$

The two cases are relevant:

(i)

$$\begin{aligned} &n_Q + n - r_X > n + n_Q - r \\ &\rho_r[M(X + MQ)] = n + n_Q - (n_Q + n - r_X) = r_X \end{aligned} \tag{A.20}$$

Because X is proper $r_X > 0$, and this is unacceptable.

(ii)

$$\begin{aligned} &n_Q + n - r_X < n + n_Q - r \\ &\rho_r[M(X + MQ)] = n + n_Q - \left(n + n_Q - r\right) = r \end{aligned} \tag{A.21}$$

The requirement $\rho_r[M(X + MQ)] \in \{-1, 0\}$ means that $\rho_r(Q) = r \in \{-1, 0\}$. Thus, the relative degree of Q must be -1 or 0 to make ASPR any strictly proper plant.

□

Parallel feedforward synthesis

Theorems 3, 4, and 6 enable the synthesis of a parallel feedforward, which guarantees that the augmented plant $P + D$ is ASPR. Based on these theorems, the synthesizing algorithms are detailed as follows.

Algorithm 1 (for a proper and stable plant $P(s)$)
Select a transfer function $Q^{-1}(s)$ such that $Q^{-1}(s)$:

(i) has relative degree 0 or 1 (pole excess of 0 or 1);
(ii) is minimum phase (all zeros are in the LHP);
(iii) has no poles on the imaginary axis; and
(iv) has small DC gain;

then,

$$D(s) = Q^{-1}(s) - P(s) \tag{A.22}$$

Algorithm 2 (for a proper and stable plant $P(s)$)
Design a stabilizing controller $C(s)$ such that:

(i) $C(s)$ is stable;
(ii) $C(s)$ is minimum phase (all zeros are in the LHP);
(iii) $C(s)$ has high gain; and
(iv) $C(s)$ has relative degree 0 or -1;

then,

$$D(s) = C^{-1}(s) \tag{A.23}$$

Algorithm 3 (for a strictly proper plant $P(s)$)
Select a stable $Q(s)$ with relative degree $\{0, -1\}$. Then,

$$D^{-1} = (X + MQ)/(Y - NQ) \tag{A.24}$$

where

$$NX + MY = 1 \tag{A.25}$$

$$P(s) = C(sI - A)^{-1}B \tag{A.26}$$

and

$$M(s) = F\,(sI - (A + BF))^{-1}\,B + 1 \tag{A.27}$$

$$N(s) = C\,(sI - (A + BF))^{-1}\,B \tag{A.28}$$

$$X(s) = F\,(sI - (A + HC))^{-1}\,H \tag{A.29}$$

$$Y(s) = -F\,(sI - (A + HC))^{-1}\,B + 1 \tag{A.30}$$

are stable transfer functions. The matrices $F \in R^{1xn}$, $H \in R^{nx1}$ are selected such that $A + BF$ and $A + HC$ are stable.

Remarks

1. The SAC algorithm controls the augmented error $e_a(t)$. In order to reduce the difference $|e_a(t)-e(t)|$, the gain of $D(s)$ is required to be as small as possible. Note that low gain of $D(s)$ means high gain of $C(s)$.
2. Algorithms 1 and 3 require the use of the plant model in terms of transfer functions or state space matrices. Algorithm 2 may be applied even when only the measurements of the plant transfer function are available. In this case, the controller can be designed, for example, by using the QFT method [20].

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